PHYSICAL REVIEW B **80**, 153305 (2009)

Positive current noise cross correlations in capacitively coupled double quantum dots with ferromagnetic leads

Bing Dong and X. L. Lei
Department of Physics, Shanghai Jiaotong University, 800 Dongchuan Road, Shanghai 200240, China

N. J. M. Horing
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA
(Received 11 June 2009; published 12 October 2009)

We examine cross correlations (CCs) in the tunneling currents through two parallel interacting quantum dots coupled to four independent ferromagnetic electrodes. We find that when either one of the two circuits is in the parallel configuration with sufficiently strong polarization strength, a mechanism of dynamical spin blockade, i.e., a spin-dependent bunching of tunneling events, governs transport through the system together with the interdot Coulomb interaction, leading to a sign reversal of the zero-frequency current CC in the dynamical channel blockade regime, and to enhancement of positive current CC in the dynamical channel antiblockade regimes, in contrast to the corresponding results for the case of paramagnetic leads.

DOI: 10.1103/PhysRevB.80.153305 PACS number(s): 72.70.+m, 72.25.Rb, 73.23.Hk, 73.63.−b

Quantum noise cross correlation (CC) in multiterminal mesoscopic devices far from equilibrium has recently become an active issue because it characterizes the degree of correlation and the statistics of the charge carriers.[1,2] It is believed that in a noninteracting system, the current CC between different normal-metallic leads is always negative due to the fermionic statistics of electrons;[3] this has been confirmed experimentally in a Hanbury Brown-Twiss setup.[4] On the other hand, much theoretical work has predicted that current CC may become positive in the following situations: for a hybrid superconductor-normal system;[5] for a system with three-terminal ferromagnetic leads;[6] also for a system in the Coulomb-interaction-mediated regime;[7] and as a result of feedback effects of external voltage fluctuations.[8]

Very recently, Coulomb-interaction-induced positive current CC was experimentally observed in a double quantum-dot (QD) system, with two parallel QDs coupled via an interdot Coulomb interaction, and also coupled to four independent electrodes and two independent gates, as shown in Fig. 1.[9] It was found that when the system is biased on the specific occupation parameters, (1,0) and (0,1) [(M,N) denotes the electron numbers in the top and bottom QDs], by means of tuning two gate voltages, the electrons occupying one dot during sequential tunneling through that QD can enhance either the tunneling-in rate or the tunneling-out rate of the other dot due to the Coulomb interaction (dynamical Coulomb antiblockade), resulting in the appearance of a positive CC between the currents through the two QDs (i.e., electron bunching).[9,10] However, the current CC still remains negative in the transport regimes, (0,0) and (1,1), due to the dynamical Coulomb blockade effect (electron antibunching).

In this Brief Report, we analyze the current CC for the same structure as the one studied in Refs. 9 and 10 but with ferromagnetic leads. Experiments with a strongly interacting QD and ferromagnetic contacts have recently been performed successfully,[11] and it is not difficult to make such a structure with parallel-coupled QDs connected to four independent ferromagnetic leads, as shown in Fig. 1. Our main finding is that sufficiently polarized contacts can lead to positive current CC even in the Coulomb blockaded region (0,0) if only one of the two circuits is in the parallel configuration. Similar to results for the three-terminal QD with ferromagnetic electrodes studied by Cottet *et al.*,[6] the present result also stems from dynamical spin blockade, associated with dynamical channel blockade. Furthermore, we find that the sign of current CC reduces to the result for paramagnetic electrodes with increasing spin-relaxation rate.

The system we study here consists of two parallel single-level QDs [top and bottom $(i=t,b)$] with energies ε_i and interdot Coulomb interaction U, which are coupled to four collinearly spin-polarized leads with net spin-independent tunneling rates γ_i and polarization strengths p_i $(0\leq p_i<1)$ (Fig. 1). Accordingly, the ferromagnetism of the leads in the top/bottom circuit can be accounted for by the spin-dependent tunneling rates: $\Gamma_{Li\uparrow}=\Gamma_{Ri\uparrow}=\gamma_i(1+p_i)$ and $\Gamma_{Li\downarrow}=\Gamma_{Ri\downarrow}=\gamma_i(1-p_i)$ for the parallel (P) configuration; $\Gamma_{Li\uparrow}=\Gamma_{Ri\downarrow}=\gamma_i(1+p_i)$ and $\Gamma_{Li\downarrow}=\Gamma_{Ri\uparrow}=\gamma_i(1-p_i)$ for the antiparallel (AP) configuration. Moreover, a constant spin-flip scattering rate, γ_{sf}, is introduced to model spin relaxation. We also assume infinite on-site Coulomb repulsion to guarantee no

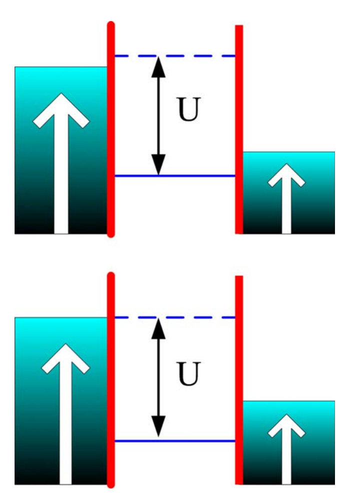

FIG. 1. (Color online) Diagram of the system.

 ©2009 The American Physical Society

double occupation on each dot. Therefore, there is a total of nine states in this system: no electron in the two QDs, $|00\rangle$, the top QD occupied by one electron with spin σ, $|\sigma 0\rangle$, the bottom QD occupied by one electron with spin σ, $|0\sigma\rangle$, and either QD occupied by one electron with spin σ (σ'), $|\sigma\sigma'\rangle$. In the sequential-tunneling limit, electronic transport through such double QDs can be described by the rate equation,[6,9,12]

$$\frac{d}{dt}\rho_{\alpha\beta} = \sum_{\alpha'\beta'} M_{\alpha\beta,\alpha'\beta'}\rho_{\alpha'\beta'}, \quad (\alpha,\beta,\alpha',\beta' = 0,\uparrow,\downarrow), \quad (1)$$

in which $\rho_{\alpha\beta}$ is the occupation probability for the state $|\alpha\beta\rangle$. The term $M_{\alpha\beta,\alpha\beta}=\Sigma_{\alpha'\beta'}M_{\alpha'\beta',\alpha\beta}$ gives the total loss rate for the state $|\alpha\beta\rangle$ while the term $M_{\alpha\beta,\alpha'\beta'}(\alpha\beta\neq\alpha'\beta')$ gives the total rate for transitions between the two states $|\alpha\beta\rangle$ and $|\alpha'\beta'\rangle$. All these terms depend on the spin-flip transition rate and the spin-dependent rates for tunneling-in and tunneling-out between a QD i and its lead ηi ($\eta=L,R$) when the other QD is either empty or occupied, $\Gamma^{\pm}_{\eta i\sigma}=\Gamma_{\eta i\sigma}f^{\pm}_{\eta i}(\epsilon_i)$ and $\tilde{\Gamma}^{\pm}_{\eta i\sigma}=\Gamma_{\eta i\sigma}f^{\pm}_{\eta i}(\epsilon_i+U)$, where $f^{+}_{\eta i}(\epsilon)=\{1+\exp[(\epsilon-\mu_{\eta i})/k_BT]\}^{-1}$ is the Fermi function of lead ηi with chemical potential $\mu_{\eta i}$ and temperature T, and $f^{-}_{\eta i}(\epsilon_i)=1-f^{+}_{\eta i}(\epsilon_i)$. For example, the total loss rates for the states $|\sigma 0\rangle$ and $|\sigma\sigma'\rangle$ are $M_{\sigma 0,\sigma 0}=-\Sigma_\eta(\Gamma^{-}_{\eta t\sigma}+\tilde{\Gamma}^{+}_{\eta b\uparrow}+\tilde{\Gamma}^{+}_{\eta b\downarrow})-\gamma_{sf}$ and $M_{\sigma\sigma',\sigma\sigma'}=-\Sigma_\eta(\tilde{\Gamma}^{-}_{\eta t\sigma}+\tilde{\Gamma}^{-}_{\eta b\sigma'})-2\gamma_{sf}$, respectively. Here, a symmetric bias voltage V_i is assumed to be applied between the left and right leads: $\mu_{Li}=-\mu_{Ri}=eV_i/2$.

The steady-state value of $\rho^0_{\alpha\beta}$ under finite bias voltage V_i is obtained by solving $d\rho_{\alpha\beta}/dt=0$ in Eq. (1). Then the spin-dependent currents flowing through the right leads can be evaluated as (we use $\hbar=e=k_B=1$)

$$I_{t\uparrow}(t) = \Gamma^{-}_{Rt\uparrow}\rho_{\uparrow 0} - \Gamma^{+}_{Rt\uparrow}\rho_{00} + \tilde{\Gamma}^{-}_{Rt\uparrow}(\rho_{\uparrow\uparrow}+\rho_{\uparrow\downarrow}) - \tilde{\Gamma}^{+}_{Rt\uparrow}(\rho_{0\uparrow}+\rho_{0\downarrow}), \quad (2)$$

$$I_{b\uparrow}(t) = \Gamma^{-}_{Rb\uparrow}\rho_{0\uparrow} - \Gamma^{+}_{Rb\uparrow}\rho_{00} + \tilde{\Gamma}^{-}_{Rb\uparrow}(\rho_{\uparrow\uparrow}+\rho_{\downarrow\uparrow}) - \tilde{\Gamma}^{+}_{Rb\uparrow}(\rho_{\uparrow 0}+\rho_{\downarrow 0}), \quad (3)$$

and $I_{t\downarrow}$ and $I_{b\downarrow}$ are obtained by interchanging $\uparrow\leftrightarrow\downarrow$ in Eqs. (2) and (3), respectively. Using the techniques developed in Refs. 6, 9, 13, and 14, we introduce spin-resolved current matrices for the two circuits according to Eqs. (2) and (3), and then apply them to the steady-state solutions of Eq. (1) to calculate the average currents $I_{t(b)}=\Sigma_\sigma I_{t(b)\sigma}$ and the spin-dependent noise CC $S_{t\sigma,b\sigma'}(\omega)$ defined as

$$S_{t\sigma,b\sigma'}(\omega) = 2\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau}\langle\Delta I_{t\sigma}(\tau)\Delta I_{b\sigma'}(0)\rangle, \quad (4)$$

with $\Delta I_{i\sigma}(t)=I_{i\sigma}(t)-\langle I_{i\sigma}\rangle$. The total zero-frequency current CC is $S_{tb}=\Sigma_{\sigma\sigma'}S_{t\sigma,b\sigma'}(0)$.

In the following calculation, we focus on the situation of two identical circuits, $p_t=p_b=p$ and $\gamma_t=\gamma_b=\gamma$. The other parameters are: $U=6\gamma$, $T=\gamma/2$ and a given bias voltage $V_t=V_b=V=10\gamma$ (hereafter, in the text and in the figures we use γ as the energy unit). Figure 2 exhibits the calculated zero-frequency current CC dependence on the energy levels of the two QDs, ε_t and ε_b, for paramagnetic leads (a), and for ferromagnetic leads with strong polarization, $p=0.6$, and no spin-flip scattering $\gamma_{sf}=0$ (b)–(d). In the paramagnetic case, the current CC is, as expected, negative for the occupation regimes, (0,0) and (1,1), while it is positive for the regimes, (1,0) and (0,1). It is already known that this positive CC stems from interaction-induced enhancement of the tunneling rate.[9] To reveal this with greater clarity, we have derived an analytic expression for the current CC for the regime (1,0) (for example, setting $\varepsilon_t=-5.5$ and $\varepsilon_b=0$) in the zero temperature by assuming an enhanced tunneling-in rate for the bottom QD under the condition that the top QD is occupied by an electron, $\tilde{\Gamma}^{+}_{Lb\sigma}=x(0<x\leq 1/2)$ (correspondingly, the tunneling-out rate is $\tilde{\Gamma}^{-}_{Lb\sigma}=1-x$; the other rates are either 1 or 0).[15] The analytic result is given by

$$S_{tb} = -\frac{2}{27}\gamma x\frac{89x^2+2813x-1890}{(5x+6)^3} > 0. \quad (5)$$

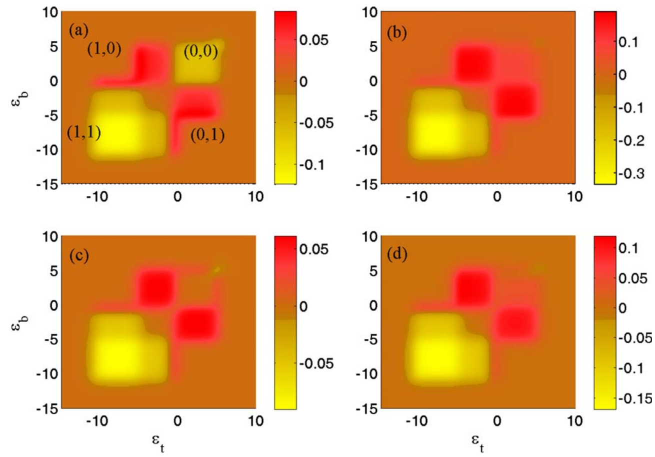

FIG. 2. (Color online) Zero-frequency cross correlation dependence on both gate voltages for transport bias voltage $V=10\gamma$. (a) is plotted for paramagnetic leads; (b)–(d) are for ferromagnetic leads with $p=0.6$ and without spin-flip scattering $\gamma_{sf}=0$. Both circuits are in the P configuration (b), the AP configuration (c), and one circuit is in the P configuration and the other one is in the AP configuration (d). The numbers (M,N) in (a) mean the different occupation regimes.

Surprisingly, we find that the sign of current CC for the transport regime (0,0) is changed in the presence of ferromagnetic electrodes [Figs. 2(b)–2(d)] and its largest values are obtained in the case when both two circuits are in the P configuration (P-P configuration). Furthermore, the positive CC noise for the transport regimes, (1,0) and (0,1), is enhanced in comparison with those in the case of paramagnetic leads, if either of the two circuits is in the P configuration. This sign reversal and the enhancement can be explained in terms of the associated effect of dynamical channel blockade and the partitioning of the electrons into fast movers and slow movers due to spin-polarized electrodes.

Let us focus on the P-P configuration. Here, we assume that up spins are in the majority. Correspondingly, the dwell time of down spins on the dot is longer than that of up spins since the down-spin tunneling rates (slow electrons) are much lower than those of up spins (fast electrons). Considering further the fact that the total number of electrons with up spin, $n_{i\uparrow}$, is equal to that of electrons with down spin, $n_{i\downarrow}$, in this configuration, we can infer that every tunneling event of a down-spin electron flowing through a QD must follow several consecutive tunneling events of up-spin electrons flowing through the QD, i.e., up spins bunching on both circuits. This indicates that for most time intervals, the sign of $\Delta I_{t\uparrow}(\tau)$ is the same as that of $\Delta I_{b\uparrow}(0)$, but is opposite to that of $\Delta I_{b\downarrow}(0)$ due to the partitioning effect, which is responsible for the occurrence of positive CC between up spins, $S_{t\uparrow,b\uparrow}(0)$; but negative CC involves down spins, $S_{t\uparrow,b\downarrow}(0)$, for the transport regimes, (0,0), (1,0), and (0,1). We have obtained analytical expressions for the various spin-resolved CCs as

$$S_{t\uparrow,b\uparrow}(0) = -\frac{2}{125}\gamma(1+p)^2\frac{\gamma(1-5p)+2\gamma_{sf}}{2\gamma_{sf}+\gamma(1-p^2)}, \quad (6)$$

$$S_{t\downarrow,b\downarrow}(0) = -\frac{2}{125}\gamma(1-p)^2\frac{\gamma(1+5p)+2\gamma_{sf}}{2\gamma_{sf}+\gamma(1-p^2)}, \quad (7)$$

$$S_{t\uparrow,b\downarrow}(0) = S_{t\downarrow,b\uparrow}(0) = \frac{-2\gamma}{125}(1-p^2)\frac{\gamma+2\gamma_{sf}}{2\gamma_{sf}+\gamma(1-p^2)}, \quad (8)$$

and for the total CC as

$$S_{tb}(0) = -\frac{8}{125}\gamma\frac{\gamma(1-5p^2)+2\gamma_{sf}}{2\gamma_{sf}+\gamma(1-p^2)}, \quad (9)$$

in the transport regime (0,0) at zero temperature. Clearly, the positive sign of $S_{tb}(0)$ stems from the up-up CC $S_{t\uparrow,b\uparrow}(0) > 0$ for sufficiently strong polarization $p > 1/\sqrt{5} \approx 0.45$ in absence of spin-flip scattering $\gamma_{sf}=0$ (note that the other spin-resolved CCs are all negative). Moreover, retaining spin coherence during tunneling is a necessary condition for such bunching of up spins. We exhibit the effect of spin-flip scattering on the zero-frequency current CCs in Fig. 3. It should be noted that spin-flip scattering strongly influences the up-up CC once γ_{sf} is comparable to the tunneling rate γ; and in the limit of infinitely high γ_{sf}, the CCs tend to the corresponding values of the paramagnetic case for any value of the polarization and any configuration [see Eq. (9)]. In particular, $S_{tb}(0)$ becomes negative around $\gamma_{sf} \simeq 2\gamma$ in Fig. 3(a). Moreover, a weak polarization, p, cannot cause sufficiently strong up-up spin bunching to overcome the negative CCs, $S_{t\sigma,b\downarrow}(0)$ and $S_{t\downarrow,b\sigma}(0)$, leading to a small negative value of $S_{tb}(0)$, as shown in Fig. 3(d) for the results with $p=0.3$.

Figure 3(b) exhibits the results for the transport regime (1,0). It is obvious that the spin-resolved CCs are all positive due to dynamical channel antiblockade; moreover, the dynamical spin blockade strongly enhances $S_{t\uparrow,b\uparrow}(0)$ and weakly suppresses $S_{t\downarrow,b\downarrow}(0)$ and $S_{t\downarrow,b\uparrow}(0)$ in comparison with the results of the paramagnetic case. Therefore, the two effects are positively additive and they result in an enhanced positive CC for the transport regimes, (1,0) and (0,1). For the double occupation case (1,1), interdot Coulomb blockade is the dominant mechanism governing transport (over the dynamical spin blockade), giving rise to a negative value, but enhanced in magnitude, for the up-up CC [Fig. 3(c)].

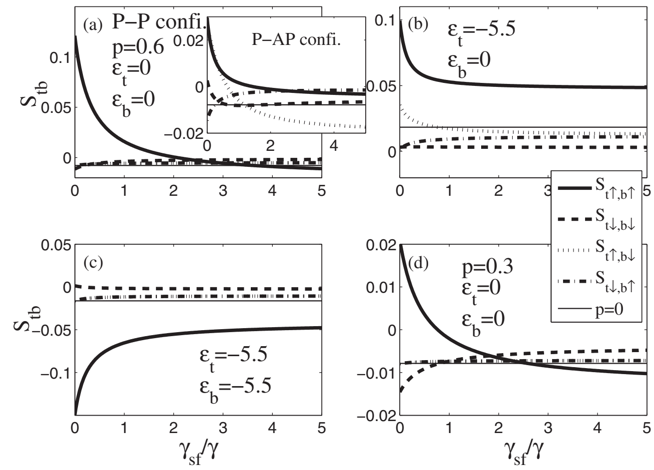

FIG. 3. Spin-resolved noise cross correlations as a function of spin-flip scattering, γ_{sf}, for the case of ferromagnetic leads in the P-P configuration. (a)–(c) are plotted for $p=0.6$ (solid lines), $p=0$ (thin line), and the chosen values of energy levels of the two QDs corresponding to the various transport regimes, (0,0), (1,0), and (1,1), respectively; (d) is plotted for $p=0.3$ in the transport regime (0,0). The inset in (a) shows the corresponding results for the transport regime (0,0) with $p=0.6$ in the P-AP configuration.

On the contrary, the situation is quite different when one of the circuits is in the AP configuration. For the AP case, the fast electron has spin up at the input terminal but has spin down at the output terminal, leading to an accumulation of up spins and a rather weak bunching of tunneling events associated with down spin at the output terminal. When the top circuit is in the P configuration while the bottom circuit is in the AP configuration (P-AP configuration), we find very small positive values for $S_{t\uparrow,b\uparrow}(0)$ and $S_{t\uparrow,b\downarrow}(0)$, and thus a vanishing $S_{tb}(0)$ for the transport regime (0,0) [inset of Figs. 2(d) and 3(a)]. A similar result is obtained when both circuits are in the AP configuration [Fig. 2(c)].

Our results show that a parallel-coupled QD in the P-P configuration is optimal for the occurrence of positive current CC: in the Coulomb antiblockade regimes, (1,0) and (0,1), the maximum value of the CC can be nearly two times higher than that of the paramagnetic case. For instance, $S_{tb}(0) \simeq 0.06\gamma$ with $p=0$ while $S_{tb}(0) \simeq 0.18\gamma$ with $p=0.6$ and $\simeq 0.1\gamma$ with $p=0.4$. In the Coulomb blockade regime, (0,0), the current CC can be $S_{tb}(0) \simeq 0.08\gamma$ for $p=0.6$ and $\simeq 0.02\gamma$ for $p=0.5$. The sign change in the current CC between different output terminals due to dynamical spin blockade has not yet been observed experimentally for the original setup, a three-terminal QD with ferromagnetic leads.[6] The main difficulty may be due to attaching the third ferromagnetic lead to the QD such that its tunnel coupling strength is comparable to that of the others. Our present proposal involves only a simpler element, an interacting QD connected to two ferromagnetic leads, which has already been realized experimentally,[11] and it may provide a more accessible setup for the observation of positive CCs originating from dynamical spin blockade.

In conclusion, we have analyzed the gate-voltagedependent CC noise between two output terminals in tunneling through two capacitively coupled QDs connected to four independent ferromagnetic electrodes. We find a sign reversal of the zero-frequency CC noise in the dynamical channel blockade regime, if the polarization of the electrodes is sufficiently strong. Moreover, in the dynamical channel anti-blockade regimes, positive CC noises are obviously enhanced in comparison with the results of paramagnetic leads. These results may be ascribed to the joint effect of dynamical Coulomb blockade and dynamical spin blockade, and are found to be quite robust against the spin-flip relaxation.

This work was supported by Projects of the National Science Foundation of China, Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) of China, and the Program for New Century Excellent Talents in University (NCET). N.J.M.H. gratefully acknowledges support by DARPA under Grant No. HR0011-09-1-0008.

[1] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).

[2] T. Martin, A. Crepieux, and N. Chtchelkatchev, in *Quantum Noise in Mesoscopic Physics*, NATO Science Series II: Mathematics, Physics and Chemistry, edited by Yu. V. Nazarov (Kluwer, Dordrecht, 2003), Vol. 97.

[3] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990); Phys. Rev. B **46**, 12485 (1992).

[4] M. Henny, S. Oberholzer, C. Strunk, T. Heinzel, K. Ensslin, M. Holland, and C. Schönenberger, Science **284**, 296 (1999); W. D. Oliver, J. Kim, R. C. Liu, and Y. Yamamoto, *ibid.* **284**, 299 (1999); H. Kiesel, A. Renz, and F. Hasselbach, Nature (London) **418**, 392 (2002); S. Oberholzer, E. Bieri, C. Schönenberger, M. Giovannini, and J. Faist, Phys. Rev. Lett. **96**, 046804 (2006).

[5] M. P. Anantram and S. Datta, Phys. Rev. B **53**, 16390 (1996); T. Martin, Phys. Lett. A **220**, 137 (1996); J. Torres and T. Martin, Eur. Phys. J. B **12**, 319 (1999); F. Taddei and R. Fazio, Phys. Rev. B **65**, 134522 (2002).

[6] A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. Lett. **92**, 206801 (2004); Phys. Rev. B **70**, 115315 (2004).

[7] A. M. Martin and M. Büttiker, Phys. Rev. Lett. **84**, 3386 (2000).

[8] C. Texier and M. Büttiker, Phys. Rev. B **62**, 7454 (2000); S.-T. Wu and S. Yip, *ibid.* **72**, 153101 (2005); V. Rychkov and M. Büttiker, Phys. Rev. Lett. **96**, 166806 (2006).

[9] D. T. McClure, L. DiCarlo, Y. Zhang, H.-A. Engel, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **98**, 056801 (2007).

[10] S. Haupt, J. Aghassi, M. H. Hettler, and G. Schön, arXiv:0802.3579 (unpublished).

[11] A. N. Pasupathy, R. C. Bialczak, J. Martinek, J. E. Grose, L. A. K. Donev, P. L. McEuen, and D. C. Ralph, Science **306**, 86 (2004); S. Sahoo, T. Kontos, J. Furer, C. Hoffmann, M. Graber, A. Cottet, and C. Schönenberger, Nat. Phys. **1**, 99 (2005); R. S. Liu, D. Suyatin, H. Pettersson, and L. Samuelson, Nanotechnology **18**, 055302 (2007); K. Hamaya, M. Kitabatake, K. Shibata, M. Jung, M. Kawamura, K. Hirakawa, T. Machida, T. Taniyama, S. Ishida, and Y. Arakawa, Appl. Phys. Lett. **91**, 232105 (2007); J. Hauptmann, J. Paaske, and P. Lindelof, Nat. Phys. **4**, 373 (2008).

[12] Bing Dong, H. L. Cui, and X. L. Lei, Phys. Rev. B **69**, 035324 (2004).

[13] A. N. Korotkov, Phys. Rev. B **49**, 10381 (1994); S. Hershfield, J. H. Davies, P. Hyldgaard, C. J. Stanton, and J. W. Wilkins, *ibid.* **47**, 1967 (1993); U. Hanke, Yu. M. Galperin, K. A. Chao, and N. Zou, *ibid.* **48**, 17209 (1993).

[14] I. Djuric, Bing Dong, and H. L. Cui, IEEE Trans. NanoTechnol. **4**, 71 (2005); Appl. Phys. Lett. **87**, 032105 (2005); J. Appl. Phys. **99**, 063710 (2006).

[15] We derive all analytic expressions, Eqs. (5)–(9), using the number-resolved type of rate equations and the MacDonald's formula. The technique details can be found in B. Dong, H. L. Cui, and X. L. Lei, Phys. Rev. Lett. **94**, 066601 (2005).